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NAVAL POSTGRADUATE SCHOOL

Monterey, California

FINITE DIFFERENCE TREATMENT OF TRANSIENT TEMPERATURE DISTRIBUTION IN AN INSULATED PIPE

by

John E. Brock

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Abstract: This report discusses a Crank-Nicolson type of finite difference formulation for the problem of transient temperature distribution in a uniform cylindrical pipe with external insulation and containing a fluid for which temperature, pressure, and flow rate are given as functions of time. All material properties are assumed to depend on temperature. Discussion is also given of a digital computer program which implements this formulation. This implementation incorporates properties of superheated steam, the internal fluid, and of 2-1/4 Cr 1 Mo (P22), the pipe material. Stress calculations are also treated.

C O N T E N T S

1. Objective

The purpose of this monograph is to present and to document a procedure for determining transient temperature distribution and the corresponding significant thermal stress components in metallic pipes which are insulated externally and which contain steam the temperature, pressure, and flow history of which is specified. Although a classical solution is known (13)* for an idealized version of this problem, it is not capable of dealing with the variable heat transfer coefficient which governs the exchange of heat between the pipe wall and the flowing steam it contains; this coefficient is a complicated function of the properties of steam. Furthermore, the analytic solution presumes constant metal thermal conductivity and thermal diffusivity while actual piping materials exhibit very significant variations of conductivity and diffusivity over ranges of temperature usually encountered in engineering practice. Accordingly a numerical approach, capable of dealing with these variations, has been chosen for the procedure.

2. Derivation of the governing nonlinear partial differential equation

A convenient point of departure for the derivation of the governing nonlinear partial differential equation is the relation (5)

$$\lambda\mu\nu\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial \xi}\left(\frac{\mu\nu}{\lambda}k\frac{\partial T}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{\nu\lambda}{\mu}k\frac{\partial T}{\partial \eta}\right) + \frac{\partial}{\partial \zeta}\left(\frac{\lambda\mu}{\nu}k\frac{\partial T}{\partial \zeta}\right) \qquad (1)$$

Here and later T denotes temperature, t denotes time, c denotes spec-

* References are listed on pages 28 and 29.

ific heat at constant pressure, and ρ denotes mass per unit volume. Here ξ, η, and ζ denote generalized space coordinates and λ, μ, and ν are coefficients such that the distance ds between points (ξ,η,ζ) and $(\xi+d\xi,\eta+d\eta,\zeta+d\zeta)$ is given by

$$(ds)^2 = \lambda^2(d\xi)^2 + \mu^2(d\eta)^2 + \nu^2(d\zeta)^2 \qquad (2)$$

Specializing to the cylindrical coordinate system, we take $\xi = r$, $\eta = \theta$, $\zeta = z$. Furthermore, we assume axial symmetry so that $\frac{\partial T}{\partial \theta} = 0$, and we presume no variation with respect to axial position so that $\frac{\partial T}{\partial z} = 0$. We thus easily arrive at the evaluations $\lambda = \nu = 1$, $\mu = r$, and at the equation

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{k}\frac{\partial k}{\partial r}\frac{\partial T}{\partial r} \qquad (3)$$

where $\alpha = k/\rho c$ is the diffusivity. The variation of k with respect to r is due solely to the fact that conductivity is a function of temperature which itself depends on radius. Thus we finally arrive at

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \left(\frac{k'}{k}\right)\left(\frac{\partial T}{\partial r}\right)^2 \qquad (4)$$

where $k' = dk/dT$ for the pipe material. If the variation of k with respect to T were simple, there would be some point in dealing with the Kirchhoff transformation (6). However, it is not, and we will deal directly with the nonlinear partial differential equation 4.

3. Finite difference formulation

We employ a Crank-Nicolson finite difference formulation, as follows. Equation 4 is differenced in time according to

$$[(\P T)^+ + (\P T)]/2 = (T^+ - T)/[\delta(\alpha^+ + \alpha)/2] \qquad (5)$$

where δ represents the time increment, superscript $(^+)$ indicates evaluation at the advanced time $t + \delta$, and $\P$ is the space operation

$$\P T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + (k'/k)\left(\frac{\partial T}{\partial r}\right)^2 \qquad (6)$$

The latter is next represented approximately by a finite difference operation. We use the notations

$$\Delta = \text{radius increment},\ A = \left.\frac{\partial^2 T}{\partial r^2}\right|_{r_i},\ B = \left.\frac{\partial T}{\partial r}\right|_{r_i},\ \rho = k'/k$$

(Note the new use of symbol ρ.) Then, generally,

$$T_{i+1} = T_i + B\Delta + A\Delta^2/2;\quad T_{i-1} = T_i - B\Delta + A\Delta^2/2 \qquad (7)$$

where subscript $(_i)$ indicates evaluation at $r = r_i = n_i$, subscript $(_{i-1})$ indicates evaluation at $r = r_{i-1} = (n_i - 1)$, etc. (In general n_i will not be an integer.) We easily find

$$A = (T_{i+1} + T_{i-1} - 2T_i)/\Delta^2;\quad B = (T_{i+1} - T_{i-1})/2\Delta \qquad (8)$$

so that

$$\P T = \{T_{i+1} + T_{i-1} - 2T_i + (T_{i+1} - T_{i-1})[1/2n_i + \rho_i(T_{i+1} - T_{i-1})/4]\}/\Delta^2 \qquad (9)$$

Setting $\beta_i = 4\Delta^2/\delta(\alpha_i^+ + \alpha_i)$, we arrange the general equation in the form

$$\left[-1 + \frac{1}{2n_i} + \rho_i^+(T_{i+1}^+ - T_{i-1}^+)/4\right]T_{i-1}^+ + (\beta_i + 2)T_i^+ - \left[1 + \frac{1}{2n_i} + \rho_i^+(T_{i+1}^+ - T_{i-1}^+)/4\right]T_{i+1}^+$$

$$= (\beta_i - 2)T_i + T_{i+1} + T_{i-1} + (T_{i+1} - T_{i-1})\left[\frac{1}{2n_i} + \rho_i(T_{i+1} - T_{i-1})/4\right] \qquad (10)$$

On the left, the bracketed coefficients of the advanced temperatures T_{i-1}^+ and T_{i+1}^+ themselves contain these (unknown) advanced temperatures. Accordingly, it is contemplated that iteration must be used in which earlier evaluations of T_{i-1}^+ and T_{i+1}^+ are used in evaluating the bracketed coefficients.

Equation 10 is valid for interior nodal points, $i = 2,3,\ldots,N$, where N is the number of subdivisions of the pipe wall. At the inside surface where the radius is $r_1 = n_1\Delta$, there is a convective boundary condition

$$k_1 \frac{\partial T}{\partial r} = h(T_1 - \phi) \tag{11}$$

where ϕ is the fluid temperature and h is the surface heat transfer coefficient. We write this as

$$\left.\frac{\partial T}{\partial r}\right|_{r_1} = B = (\sigma/\Delta)(T_1 - \phi) \tag{12}$$

where $\sigma = h\Delta/k_1$, and we also use the first of equations 7, taking i = 1. Thus we get

$$\P T = \{2(T_2 - T_1) - \sigma(T_1 - \phi)[2 - \frac{1}{n_1} - \rho_1\sigma(T_1 - \phi)]\}/\Delta^2 \tag{13}$$

Thus, the appropriate equation for the inner surface is

$$\{\beta_1 + 2 + \sigma^+[2 - \frac{1}{n_1} - \rho_1^+\sigma^+(T_1^+ - 2\phi^+)]\}T_1^+ - 2T_2^+$$
$$= (\beta_1 - 2)T_1 + 2T_2 - \sigma(T_1 - \phi)[2 - \frac{1}{n_1} - \rho_1\sigma(T_1 - \phi)]$$
$$+ \sigma^+\phi^+(2 - \frac{1}{n_1} + \rho_1^+\sigma^+\phi^+) \tag{14}$$

where $\beta_1 = 4\Delta^2/\delta(\alpha_1^+ + \alpha_1)$. Again, note the nonlinearity represented by the appearance of the advanced temperatures in the bracketed coefficients and on the right side. The advanced temperature also appears in β_1 and in σ^+ since α_1^+ and h depend on the advanced inside surface metal temperature.

Finally, the last equation pertains to the exterior surface node (numbered M = N+1) where there is perfect insulation so that $\frac{\partial T}{\partial r} = 0$. Employing also the second of equations 7 with i = M, we get

$$\P T = 2(T_N - T_M)/\Delta^2 \tag{15}$$

and the last difference equation becomes

$$(\beta_M + 2)T_M^+ - 2T_N^+ = (\beta_M - 2)T_M + 2T_N \tag{16}$$

4. Method of solution

Because of the nonlinearities, an iterative solution is used. However, since the coefficient matrix is tridiagonal, it is convenient and expeditious to use a pivotal method several times rather than to

use a purely iterative method such as that of Gauss-Seidel. We may think of the equations in the form (cf. Reference 14)

$$a_i T^+_{i-1} + b_i T^+_i + c_i T^+_{i+1} = d_i \quad (i=1,2,\ldots,N,M) \qquad (17)$$

with $a_1 = c_M = 0$. Then we form

$$c^*_0 = 0; \quad c^*_i = c_i/(b_i - a_i c^*_{i-1}) \quad (i=2,\ldots,N) \qquad (18a)$$

$$d^*_0 = 0; \quad d^*_i = (d_i - a_i d^*_{i-1})/(b_i - a_i c^*_{i-1}) \quad (i=1,2,\ldots,M) \qquad (18b)$$

and obtain the advanced temperatures as

$$T^+_M = d^*_M; \quad T^+_i = d^*_i - c^*_i T^+_{i+1} \quad (i=N,N-1,\ldots,2.1) \qquad (19)$$

Because of the appearance of the advanced temperatures in the coefficients, several iterations are required.

5. Digital computer program.

The digital computer program discussed in appendix C hereto implements the theory developed above. The main program is called PIPETEM. The properties of the fluid are determined by subroutines CSUBP, PRAND, and ABSMU. These are called by subroutine AITCH which determines the surface heat transfer coefficient h. The only fluid whose properties are incorporated in the present program is superheated steam. The properties of the metallic piping material are determined by subroutines METAL, COEXP, and EYUNG. All these subroutines will be discussed further in appendix A hereto.

6. Properties of the solution.

Having determined the advanced temperatures T^+_i, (i=1,...,M), certain significant properties of this solution are determined in a subroutine called AVERJ. These properties are TAV, DT1, DT2, SL, and SR which will be defined and described at this point.

TAV, DT1, and DT2 are rigorous counterparts to "average temperature," "delta-tee-one," and "delta-tee-two" given in various piping and vessel codes for the estimation of thermal stress. As will be pointed out in appendix B, for uniform cylinders with axial symmetry of loading and temperature, the proper stresses to be used in code evaluations may be determined directly without the use of these quantities; however, their use is widespread and they are calculated for comparison purposes. The present writer is responsible for the definitions given in the codes. For simplicity, these definitions were based on flat plate geometry and this is quite reasonable for pipes except those having unusually thick walls. That is, in these definitions, pipe diameter was assumed to be quite large comparted to pipe wall thickness. In subroutine AVERJ this assumption is not made. The appropriate definitions, based on truly cylindrical geometry, are given below, followed by demonstrations that they reduce to the code definitions as pipe radius goes to infinity while maintaining constant wall thickness.

TAV, also designated as $\overline{T}$, is the average metal temperature defined as

$$\overline{T} = \int TdA/\int dA = 2\pi\int_a^b Trdr/2\pi\int_a^b rdr = (1/hc)\int_a^b Trdr \qquad (20)$$

where a = inside radius, b = outside radius, h = b-a = wall thickness (do not confuse with earlier use of h as heat transfer coefficient), and c = (b+a)/2 = mean radius. If we define y = r-c, then

$$\overline{T} = (1/h)\int_{-h/2}^{+h/2}(1+y/c)Tdy \to (1/h)\int_{-h/2}^{+h/2} Tdy \text{ as } c \to \infty \qquad (21)$$

and this is the definition given in the codes.

We next define a quantity which can be termed the thermal moment, viz.:

$$M = \int r(T-\bar{T})dA = 2\pi\int_a^b r^2(T-\bar{T})dr = 2\pi c^2\int_{-h/2}^{+h/2}(1+y/c)^2(T-\bar{T})dy \qquad (22)$$

Using the definition of $\bar{T}$, this can be simplified to the form

$$M = 2\pi c\int_{-h/2}^{+h/2}(1+y/c)Tydy - \pi\bar{T}h^3/6 \qquad (23)$$

The next step in this development is to consider the average and the thermal moment of a linearly varying temperature distribution which has variation V from r = a to r = b. Such a temperature is given by T = Vr/h for which we find

$$\bar{T} = (V/h^2c)\int_a^b r^2dr = (V/3h^2c)(b^3-a^3) \qquad (24)$$

$$M = 2\pi\int_a^b r^2(Vr/h-\bar{T})dr = (\pi Vch^2/6)(1-h^2/12c^2) \qquad (25)$$

It requires considerable manipulation to obtain the last form.

By equating these two values of M, one evaluates the (equivalent) linear variation

$$V = [12\int_{-h/2}^{+h/2}(1+y/c)Tydy - \bar{T}h^3/c]/h^2(1-h^2/12c^2) \rightarrow (12/h^2)\int_{-h/2}^{+h/2}Tydy \qquad (26)$$

as $c \rightarrow \infty$. The limiting values, as $c \rightarrow \infty$, are, in each case, the definitions given by the various piping and vessel codes. The present subroutine, AVERJ, employs the more general definitions based on cylindrical geometry with finite radii. The variation V is also denotes as ΔT_1 in the codes and as DT1 in the output of the present program.

In the codes the quantity ΔT_2 is defined as the nonlinear residual temperature at the inner or the outer radius (whichever gives the greater result) after the constant part, $\bar{T}$, and the linearly varying part, V, are subtracted. In the output of the present program this quantity is denoted as DT2 and defined as

$$DT2 = Max\{|T_M-\bar{T}-DT1/2|,|T_1-\bar{T}+DT1/2|\} \qquad (27)$$

The quantities ΔT_1 and ΔT_2 are intended, in the codes, to permit estimating significant thermal stresses in general pressure vessel components. However, in the particular case of uniform hollow cylinders, the stresses resulting from radial temperature gradient are definitely classified as "local thermal stresses" and there is no actual need to separate the actual temperature distribution as has been indicated above. Instead, the thermal stresses can be calculated directly, and this is done in subroutine AVERJ. The quantities SL and SR are the values of circumferential (or axial -- they are equal) stress due to the temperature distribution at inside and outside metal surfaces, respectively, under the assumption of zero net axial force and bending and twisting moment. The latter contributions may be determined separately by use of a conventional piping flexibility analysis.

Because of the variability (with temperature) of the Young's modulus and of the coefficient of thermal expansion, the calculations are not trivial. Discussion of this matter is given in appendix B hereto.

APPENDIX A Discussion of the subroutines

Subroutines CSUBP, PRAND, and ABSMU receive as inputs the temperature (°F) and the pressure (psia) of the steam flowing in the pipe and produce as outputs the values of c_p (specific heat at constant pressure), N_p (Prandtl's number), and μ (absolute viscosity) of the steam. The calculations are by polynomials in T and P the coefficients of which were obtained by least squares analysis of data points read from the graphs on pages 293, 294, and 297 of the ASME Steam Tables ( ). It is implicitely assumed that the steam is saturated or superheated. If the steam is wet, the proper values are probably not obtained.

Sixty number pairs were used in dtermining the coefficients in CSUBP, fifty-four pairs for PRAND, and twenty-four pairs for ABSMU. The only type of accuracy evaluation performed was to observe that each of the subroutines was able to generate the input number pairs within one or two percent (generally much closer than this). Such accuracy is greater than that implicit in convective heat tranfer theory and certainly at least as great as the accuracy with which the input values were read from the graphs.

Subroutine AITCH determines the film heat transfer coefficient h by use of the Colburn formula (cf. Giedt (8))

$$h = N_{ST} G c_p \qquad \text{(A-1)}$$

where G is mass per unit time per unit area, i.e.,

$$G = \dot{m}/A_{flow} \qquad \text{(A-2)}$$

and N_{ST} is the Stanton number,

$$N_{ST} = 0.023\ N_{PR}^{-2/3}\ N_{RE}^{-1/5} \qquad (A-3)$$

In this formula N_{PR} is the Prandtl number and N_{RE} is the Reynolds number

$$N_{RE} = Gd/\mu \qquad (A-4)$$

where d is the pipe inside diameter. The Colburn formula is one which is recommended for cases of well developed turbulent flow at fairly high Reynolds numbers. It seems to be an appropriate choice except perhaps for cases in which the operating transient involves a very great decrease in mass flow rate. In such cases $\dot{Q}$ indeed approaches zero but not necessarily in the way represented by the Colburn formula. Incidentally, subroutine AITCH avoids a computational difficulty as G goes to zero by combining the equations in the form

$$h = 0.023\ c_p\ G^{0.8}\ (\mu/d)^{0.2}\ /\ N_{PR}^{2/3} \qquad (A-5)$$

The specific heat c_p is evaluated at the fluid bulk temperature but N_{PR} and μ are evaluated at the "film" temperature which is the arithmetic average of fluid bulk temperature and inside wall metal temperature. Since the latter is an unknown in the calculations, an iterative evaluation must be used.

Subroutines METAL, COEXP, and EYUNG obtain pertinent properties of the metal pipe material corresponding to input values of the metal temperature. These subroutines presently provide data for only one particular material, the standard low chrome-moly high temperature alloy designated by the number 22 -- pipe is designated as P22. Subroutine METAL provides values of thermal conductivity (k), the rate of variation (k' = dk/dT), and the thermal diffusivity using polynomial representations of material test data.

Although Table 1-4.0 of the ASME Nuclear Components Code (2) suggests using the same values for thermal conductivity and thermal diffusivity for P22 as for low carbon steel, data given in the TPRC Data series, References (15) and (16), indicate a sufficiently great dependence on alloying constituents that we have preferred to use the latter. Thus we get substantially lower values of conductivity and diffusivity than the ASME tabulation gives. The actual data employed in constructing subroutine METAL are the data points from curve 19 page 1158 of Ref. (15) and curve 2 page 343 of Ref. (16). It should be noted, however, that it would be a very simple matter to modify the subroutine METAL to use the ASME data rather than the TPRC data.

Subroutines COEXP and EYUNG are used (by subroutine AVERG) to provide the coefficient of thermal expansion and the Young's modulus corresponding to the given input temperature. The data is from Appendices B and C of the Power Piping Code, Ref. (3), and linear interpolation is used. Poisson's ratio is assumed to have the constant value 0.3.

All subroutines as well as the main program use English customary units, specifically inches (rather than feet) and seconds (rather than minutes or hours).

APPENDIX B Stress Calculations

Assuming what is called a "quasi-static" situation, i.e., neglecting forces associated with accelerating the mass of the pipe as its thermal expansion causes displacement of its material particles, and assuming also that the difference between adiabatic and isothermal material constants is of negligible significance, we nevertheless are faced with a nontrivial problem in determining the state of stress which results from the temperature changes discussed in the body of this report.

Two material constants, E (Young's modulus) and α(the coefficient of linear expansion -- do not confuse with thermal diffusivity) are of significance in connection with stress evaluation. Both vary with temperature. Further, as will be discussed below, α varies with stress state. A number of writers have dealt with stresses in tubes due to changes in temperature, among them Hilton (9) and Chang and Chu (7). The analysis, in effect, reduces to the following.

$$\sigma_r = [E/r^2(1-\nu)][(r^2-a^2)\, J(b)/(b^2-a^2) - J(r)] \qquad (B\text{-}1)$$

$$\sigma_\theta = [E/r^2(1-\nu)][(r^2+a^2)\, J(b)/(b^2-a^2) + J(r) - r^2\alpha T(r)] \qquad (B\text{-}2)$$

$$\sigma_z = E\varepsilon_z + [E/(1-\nu)][2\nu J(b)/(b^2-a^2) - \alpha T(r)] \qquad (B\text{-}3)$$

where

$$J(r) = \int_a^r \xi\alpha T(\xi)\,d\xi \qquad (B\text{-}4)$$

Presuming that the axial force F_z is zero, we get

$$\varepsilon_z = 2\, J(b)/(b^2-a^2) \qquad (B\text{-}5)$$

and can thus evaluate σ_z. For all temperature distributions which

result from external heating (i.e., from heat applied to the inner or outer surfaces but not generated within the wall itself) the extremal stresses will be at r = a or at r = b. (With external insulation, it is plausible that the extreme must be at r = a.) We thus have $\sigma_r = 0$ at r = a and at r = b, and we also have

$$\sigma_\theta = \sigma_z = [E/(1-\nu)][2J(b)/(b^2-a^2) - \alpha T(a)] \quad \text{at } r = a \qquad (B\text{-}6)$$

$$\sigma_\theta = \sigma_z = [E/(1-\nu)][2J(b)/(b^2-a^2) - \alpha T(b)] \quad \text{at } r = b \qquad (B\text{-}7)$$

In computing J(r) and J(b), the α under the sign of integration is the "instantaneous" value appropriate to T(r). It is (presumably) the "zero-stress" value of α which is tabulated. Obviously, however, during the temperature changes the various elements of the pipe wall are not stress-free and the question arises of how it is that we should use the zero-stress value of α. The explanation is surely not well known (this question is not even considered in any of the standard references on thermal stress analysis); the explanation which follows was given by the writer in a report, dated November 1960, entitled "Reflections on Piping Flexibility Analysis," which was intended to explain the basis of procedures incorporated in the "MDC Document" of the Mechanical Design Committee of the ANSI (then ASA) B31 Piping Code Committee.

This explanation considers a one dimensional constrained thermal expansion problem of a carbon steel bar of length L which at 70 F fits precisely between two rigid and immovable anchors; that is, at 70 F there is no stress and no gap. The temperature is increased to 300 F. One is to determine the resulting compression stress, presuming that there is no tendency to buckle laterally. As fundamental we take the

relation

$$de = d\sigma/E + \alpha dT \tag{B-8}$$

and, since, in this problem $de = 0$, we have

$$d\sigma = -E\alpha dT \tag{B-9}$$

$$\sigma_F = -\int_{70}^{300} E(T)\alpha(T)dT \tag{B-10}$$

where subscript $(_F)$ indicates the "final" condition.

Appendices B and C of ANSI B31.1-1973, Ref. ( ), give the following data for carbon steel pipe:

T	$E \cdot 10^{-6}$	$\alpha \cdot 10^{6}$	$E\alpha$
70	27.	6.07	169.35
200	27.7	6.38	176.73
300	27.4	6.60	180.84

Using trapezoidal integration, we evaluate equation (B-10) to get

$$\sigma_F = -(169.35+176.73)(130/2)+(176.73+180.84)(100/2)$$

$$= -40374 \text{ psi} \tag{B-11}$$

(This result is incorrect as we will show.)

An alternate calculation obtains the answer by considering one anchor to be removed and letting the bar expand freely (zero-stress condition) from 70 F to 300 F and then supplying sufficient compressive load to restore the bar to its original length. This calculation gives

$$\sigma_e = \int_{70}^{300} (T)dT = [(6.07+6.38)(65)+(6.38+6.60)(50)]\cdot 10^{-6}$$

$$= 1.4583 \cdot 10^{-3} \tag{B-12}$$

$$\sigma_F = -(27.4 \cdot 10^{6})(1.4583 \cdot 10^{-3}) = -39956 \text{ psi} \tag{B-13}$$

which is a slightly, but definitely, smaller value.

Now it would indeed be disturbing if, without any inelastic behavior, the final stress state were to depend on the sequence of

load-temperature application. This would indeed be a new type of thermal ratcheting. However, we will now show that the second calculation gives the correct result for any load-temperature sequence, based upon the postulate that the final stress state is independednt of sequence and depends only on material properties, the final temperature state, and the final strain. In other words, we presume that equation (B- 8) represents an exact differential so that

$$e_F = f(\sigma_F, T_F)$$

If this is the case, we have

$$\frac{\partial}{\partial T}\left(\frac{\partial e}{\partial \sigma}\right) = \frac{\partial}{\partial \sigma}\left(\frac{\partial e}{\partial T}\right) \tag{B-14}$$

and, since

$$\frac{\partial e}{\partial T} = \alpha, \quad \frac{\partial e}{\partial \sigma} = \frac{1}{E} \tag{B-15a,b}$$

we have

$$\frac{\partial \alpha}{\partial \sigma} = \frac{d}{dT}\left(\frac{1}{E}\right) \tag{B-16}$$

(we have written d rather than ∂ on the right since, by its definition, E does not depend on σ.)

Thus, if E varies with T, then α must vary with σ; i.e., $\alpha = \alpha(T,\sigma)$. The tabulated values we use are values of $\alpha(T,0)$ and these were appropriate in the second numerical calculation but not in the first. Thus, for our original problem we should integrate

$$d\sigma = -E(T)\alpha(T,\sigma)dT \tag{B-17}$$

taking into account the dependence of α upon σ. We have

$$\alpha(T,\sigma) = \alpha(T,0) + \int_0^\sigma \left(\frac{\partial \alpha}{\partial \sigma}\right)d\sigma = \alpha(T,0) + \sigma\frac{d}{dT}\left(\frac{1}{E}\right) \tag{B-18}$$

so that

$$d\sigma = -E(T)[\alpha(T,0) + (\sigma/E^2)(dE/dT)]dT$$

$$= -E(T)\alpha(T,0)dT - \sigma dE/E \qquad (B-19)$$

Thus,

$$-\alpha(T,0)dT = (Ed\sigma - \sigma dE)/E^2 = d(\sigma/E) \qquad (B-20)$$

Performing this integration, we get

$$(\sigma_F/E_F) = -\int_{70}^{300} \alpha(T,0)dT = -1.4583\cdot10^{-3}$$

$$\sigma_F = (-1.4583\cdot10^{-3})(27.4\cdot10^{6}) = -39956 \text{ psi}$$

as in the second calculation.

In the more complicated, multi-axial stress situation with which this monograph is primarily concerned, the argument takes the following form. We imagine any element (dr,rd ,dz) of the pipe to be separated from the adjacent elements and to experience stress-free thermal expansion resulting in extensional strains of the form

$$e = \int_{T_1}^{T_2} \alpha(T,0)dT \qquad (B21)$$

Then surface forces are applied to the element to cause it to assume a form such that all elements can be fitted together in the final stressed state. Since equations (B-1) through (B-7) were derived by satisfying the equilibrium and constitutive laws, they indeed describe the final state.

One final remark needs to be made concerning stress calculations for code purposes. Although the so called "simplified" methods of analysis provide for stress components corresponding to the temperature-like quantities ΔT_1 and ΔT_2, it is very important to note that in the specific case of a uniform pipe or cylindrical vessel the stress state resulting from a radial temperature gradient does not have to be separated into "secondary" and "local" parts but, instead, the actual stress may all be considered as "local." The word "local" is quite important

in considering allowable states of stress. A definitive statement to this effect is to be found on the fifth page of the ASME "Criteria" document, Ref. (4), viz:

> "A special exception to these general rules is the case of the stress due to a radial temperature gradient in a cylindrical shell. It is specifically stated in N-412(m)(2)(6) of Section III, and in 4-112(1)(2)(6) of Appendix 4 of Division 2, that this stress may be considered a local thermal stress. In reality the linear portion of this gradient can cause deformation, but it was the opinion of the Special Committee that this exception could be safely made."

Thus the stresses calculated according to equations (B-6) and (B-7) of this appendix themselves directly give the significant thermal stresses due to the radial gradient, and these are indeed <u>local</u> thermal stresses.

APPENDIX C Instructions, Data Preparation, Typical Problem

Instructions for the use of the programs and preparation of input data are given, in the form of comments, in the initial portion of the program itself. This portion of the program reads as follows:

```
Temperature distribution, etc., in uniform pipe with exterior
insulation, convectively heated by fluid contents.  The basic
procedure is a finte difference treatment of the one (space)
dimensional partial differential equation in cylindrical co-
ordinates.  Metal properties are assumed to vary with temper-
ature.  Surface heat transfer coefficient is determined by
Colburn's relationship.  The fluid content in this version is
assumed to be superheated steam and the pipe material is P22.

Input is by means of two kinds of cards.  The first card reads
OD, WT, TINIT, N, NIT, and NPROB according to FORMAT(3F10.0,3I5).
OD is the outside diameter in inches.  WT is the wall thickness
in inches.  TINIT is the uniform initial metal temperature in
degrees Fahrenheit.  (For nonuniform initial temperature, see
below.)  N is the (even) number of equal subregions into which
the wall thickness is divided.  (If input N is odd, it is round-
ed up to the next even number.)  NIT is the number of iterations
employed each time step in order to solve the nonlinear matrix
equation.  NPROB is an integer identification number.

If TINIT < -500., the program looks for nonuniform initial metal
temperature, reading enough cards according to FORMAT(7F10.0)
to provide the (N+1) required initial temperatures.
```

Subsequent cards read I, NTEES, TIM1, TIM2,PHI1, PHI2, PRES1, PRES2, FLOW1, and FLOW2 according to FORMAT(2I5,8F8.0). I is an identifying serial number which is useful for ordering the data but which is not used in the program. NTEES is the number of time subintervals into which the large time interval from time = TIM1 until time = TIM2 is to be divided. PHI1 and PHI2 are values of the fluid temperature at times TIM1 and TIM2 respectively, This temperature is assumed to vary linearly with time. Similarly for PRES (fluid pressure in psig) and FLOW (fluid flow rate in pounds per second).

The data deck for a problem terminates with a card containing 10000 in the first five positions. A number of problems, each having a data deck as described above, may be stacked. The sequence of problems is normally terminated by a card containing -1. in the first three positions.

Some thought was given to providing an option which would permit several time intervals to be calculated per output line of print. It was decided that not only would this be a needless complication but also that frequent output is appropriate and useful during those periods when, for one reason or another, small time increments are employed.

It is easily possible to make minor internal changes so that the nodal temperatures, rather than the results calculated in AVERJ, are printed. This is useful when comparing results with those obtained by other methods of calculation.

The program listing is not given here for reasons of space economy. Those who are seriously interested in installing the program should communicate directly with the writer to make appropriate arrangements. The program is written in FORTRAN using none other than quite primitive capabilities. It has been run on an IBM 360/67 requiring only 56K core for execution (both G and H compilations). For a stack of thirty two problems, involving a total of 5240 time steps, with N = 10 (10 layers) and NIT = 5 (five iterations), time requirements were as follows:

	G Compiler	H Compiler
Compile	14.38s	23.20s
Link-edit	1.87s	1.74s
Execute	14m19.48s	11m17.03s
Total	14m35.73s	11m41.97s

```
C
C   PROGRAM PIPETEM.  J. F. BROCK, JULY 1976
C
C   TEMPERATURE DISTRIBUTION, ETC., IN UNIFORM PIPE WITH EXTERIOR INSUL-
C   ATION, CONVECTIVELY HEATED BY FLUID CONTENTS.  THE BASIC PROCEDURE
C   IS FINITE DIFFERENCE TREATMENT OF THE ONE (SPACE) DIMENSIONAL
C   PARTIAL DIFFERENTIAL EQUATION IN CYLINDRICAL COORDINATES.  METAL
C   PROPERTIES ARE ASSUMED TO VARY WITH TEMPERATURE.  SURFACE HEAT
C   TRANSFER COEFFICIENT IS DETERMINED BY COLBURN'S RELATIONSHIP.
C   THE FLUID CONTENT IN THIS VERSION IS ASSUMED TO BE SUPERHEATED
C   STEAM AND THE PIPE MATERIAL IS P22.
C
C   INPUT IS BY MEANS OF TWO KINDS OF CARDS.  THE FIRST CARD READS OD,
C   WT, TINIT, N, NIT, AND NPROB ACCORDING TO FORMAT(3F10.0,3I5).
C   OD IS OUTSIDE DIAMETER IN INCHES.  WT IS WALL THICKNESS IN INCHES.
C   TINIT IS THE UNIFORM INITIAL METAL TEMPERATURE IN DEGREES FAHREN-
C   HEIT. (FOR NONUNIFORM INITIAL TEMPERATURE, SEE BELOW.)  N IS THE
C   (EVEN) NUMBER OF EQUAL SUBREGIONS INTO WHICH THE WALL THICKNESS IS
C   DIVIDED. (IF N IS ODD, IT IS ROUNDED UP TO THE NEXT EVEN NUMBER.)
C   NIT IS THE NUMBER OF ITERATIONS EMPLOYED EACH TIME STEP IN ORDER TO
C   SOLVE THE NONLINEAR MATRIX EQUATION.  NPROB IS AN INTEGER IDENTIFI-
C   CATION NUMBER.
C
C   IF TINIT < -500., THE PROGRAM LOOKS FOR NONUNIFORM INITIAL METAL
C   TEMPERATURE, READING ENOUGH CARDS ACCORDING TO FORMAT(7F10.0) TO
C   PROVIDE THE (N+1) REQUIRED INITIAL TEMPERATURES.
C
C   SUBSEQUENT CARDS READ I, NTIES, TIM1, TIM2, PHI1, PHI2, PRES1,
C   PRES2, FLOW1, AND FLOW2 ACCORDING TO FORMAT(2I5,8F8.0).  I IS AN
C   IDENTIFYING SERIAL NUMBER WHICH IS USEFUL FOR ORDERING THE DATA BUT
C   WHICH IS NOT USED IN THE PROGRAM.  NTIES IS THE NUMBER OF TIME SUB-
C   INTERVALS INTO WHICH THE LARGE TIME INTERVAL FROM TIME =TIM1 UNTIL
C   TIME = TIM2 IS TO BE DIVIDED.  PHI1 AND PHI2 ARE VALUES OF THE FLUID
C   TEMPERATURE AT TIMES TIM1 AND TIM2 RESPECTIVELY.  THIS TEMPERATURE IS
C   ASSUMED TO VARY LINEARLY WITH TIME.  SIMILARLY FOR PRES (FLUID
C   PRESSURE IN PSIG) AND FLOW (FLUID FLOW RATE IN POUNDS PER SECOND).
C
C   THE DATA DECK FOR A PROBLEM TERMINATES WITH A CARD CONTAINING 10000
C   IN THE FIRST FIVE POSITIONS.  A NUMBER OF PROBLEMS, EACH HAVING A
C   DATA DECK AS DESCRIBED ABOVE, MAY BE STACKED.  THE SEQUENCE OF
C   PROBLEMS IS NORMALLY TERMINATED BY A CARD CONTAINING -1. IN THE
C   FIRST THREE POSITIONS.
C
```

Figure C-1 Instructions for the use of PIPETEM
This material is a part of the PIPETEM program itself.

PIPEFLOW PROBLEM NUMBER [illegible]

OUTSIDE DIAMETER = 14.012 INCHES; WALL THICKNESS = 1.881 INCHES DIVIDED INTO [illegible] LAYERS

THERE ARE 5 ITERATIONS PER TIME STEP
INITIAL TEMPERATURES ARE:
AT NODE NUMBER 1 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 2 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 3 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 4 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 5 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 6 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 7 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 8 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 9 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 10 TEMPERATURE IS 960.00 DEG. F
AT NODE NUMBER 11 TEMPERATURE IS 960.00 DEG. F

ECHO OF HISTOGRAM DATA FOLLOWS:

I	NI	TIM1	TIM2	PHI1	PHI2	PRES1	PRES2	FLOW1	FLOW2
1	5	0.	10.	960.	960.	600.	600.	82.	82.
2	5	10.	13.	960.	795.	600.	600.	82.	82.
3	10	13.	18.	795.	930.	600.	600.	82.	82.
4	10	18.	23.	930.	930.	600.	600.	82.	82.
5	55	23.	50.	930.	850.	600.	600.	82.	82.
6	50	50.	100.	850.	745.	600.	600.	82.	82.
7	10	100.	150.	745.	655.	600.	600.	82.	82.
8	10	150.	200.	655.	588.	600.	600.	82.	82.
9	10	200.	250.	588.	550.	600.	600.	82.	82.
10	10	250.	300.	550.	530.	600.	600.	82.	82.
11	10	300.	350.	530.	520.	600.	600.	82.	82.
12	10	350.	400.	520.	520.	600.	600.	82.	82.

OUTPUT FOLLOWS:

TIME	FLUTEM	FLUPRES	FLUFLOW	HH	TL	TR	TAV	DT1	DT2	SL	SR
0.0	960.00	600.00	82.20	0.58	960.00	960.00	960.00	0.00	0.00	0.00	0.00
2.00	960.00	600.00	82.20	0.58	960.00	960.00	960.00	0.00	0.00	0.00	0.00
4.00	960.00	600.00	82.20	0.58	960.00	960.00	960.00	0.00	0.00	0.00	0.00
6.00	960.00	600.00	82.20	0.58	960.00	960.00	960.00	0.00	0.00	0.00	0.00
8.00	960.00	600.00	82.20	0.58	960.00	960.00	960.00	0.00	0.00	0.00	0.00
10.00	960.00	600.00	82.20	0.58	960.00	960.00	960.00	0.00	0.00	0.00	0.00
10.50	927.00	600.00	82.20	0.58	959.03	960.00	959.87	0.21	0.33	87.62	-3.39
11.00	894.00	600.00	82.20	0.58	956.40	960.00	958.86	1.83	3.64	369.87	-14.92
11.50	861.00	600.00	82.20	0.59	952.54	960.00	956.69	1.88	6.20	762.48	-34.96
12.00	828.00	600.00	82.20	0.58	947.72	960.00	953.41	3.36	10.01	1323.80	-64.49
12.50	795.00	600.00	82.20	0.58	942.13	960.00	948.87	5.25	14.32	1989.73	-104.20
13.00	808.50	600.00	82.20	0.57	937.26	960.00	944.79	7.25	17.21	2552.82	-148.38
13.50	822.00	600.00	82.20	0.57	934.23	961.00	943.35	9.04	19.69	2971.88	-189.41
14.00	835.50	600.00	82.20	0.57	932.47	961.00	943.03	10.60	20.27	3194.24	-226.53
14.50	849.00	600.00	82.20	0.57	931.62	960.00	943.76	11.91	20.13	3272.52	-259.17
15.00	862.50	600.00	82.20	0.57	931.47	960.00	944.51	12.99	19.55	3266.85	-287.13
15.50	876.00	600.00	82.20	0.57	931.85	960.00	945.31	13.83	18.54	3198.56	-310.38
16.00	889.50	600.00	82.20	0.58	932.67	960.00	946.15	14.46	17.25	3092.07	-328.96
16.50	903.00	600.00	82.20	0.58	933.85	960.00	947.02	14.88	15.72	2971.76	-343.38
17.00	916.50	600.00	82.20	0.58	935.34	960.00	947.92	15.09	14.03	2853.48	-352.53
17.50	930.00	600.00	82.20	0.58	937.11	960.00	948.86	15.12	12.20	2744.24	-357.76
18.00	933.00	600.00	82.20	0.58	938.72	960.00	949.82	15.14	11.58	2182.42	-360.89
18.50	930.00	600.00	82.20	0.58	939.87	960.00	950.79	14.96	9.44	2025.51	-363.39
19.00	930.00	600.00	82.20	0.58	940.72	960.00	951.76	14.33	8.60	1911.23	-366.12
19.50	930.00	600.00	82.20	0.58	941.36	960.00	952.72	14.31	7.96	1824.60	-368.09
20.00	930.00	600.00	82.20	0.58	941.85	960.00	953.64	14.79	7.45	1756.09	-372.13
20.50	930.00	600.00	82.20	0.58	942.24	960.00	954.65	14.79	7.05	1702.91	-376.25
21.00	930.00	600.00	82.20	0.58	942.56	960.00	955.62	14.67	6.73	1658.75	-379.61
21.50	930.00	600.00	82.20	0.58	942.81	961.00	956.58	14.63	6.49	1621.59	-382.11
22.00	930.00	600.00	82.20	0.58	943.02	957.95	956.54	14.61	6.21	1581.84	-385.71
22.50	930.00	600.00	82.20	0.58	943.20	957.95	956.50	14.59	6.01	1544.48	-387.59
23.00	928.55	600.00	82.20	0.58	943.31	959.93	956.49	14.52	5.87	1540.87	-393.32
23.50	927.09	600.00	82.20	0.58	943.31	959.93	956.42	14.51	5.83	1541.29	-397.70
24.00	925.64	600.00	82.20	0.58	943.25	959.93	956.37	14.45	5.80	1544.67	-402.52
24.50	924.18	600.00	82.20	0.58	943.12	959.93	956.32	14.41	5.84	1555.86	-407.79
25.00	922.73	600.00	82.20	0.58	942.95	959.98	956.27	14.39	5.92	1571.03	-413.38
25.50	921.27	600.00	82.20	0.58	942.74	959.97	956.21	14.41	6.02	1591.83	-419.67
26.00	919.82	600.00	82.20	0.58	942.49	959.97	956.15	15.42	6.14	1616.57	-426.06
26.50	918.36	600.00	82.20	0.58	942.22	959.96	956.08	15.46	6.28	1644.90	-432.43
27.00	916.91	600.00	82.20	0.58	941.91	959.95	956.01	15.22	6.44	1675.11	-438.16
27.50	915.45	600.00	82.20	0.58	941.59	959.93	955.94	15.45	6.61	1707.16	-447.74
28.00	914.00	600.00	82.20	0.58	941.24	959.92	955.86	15.45	6.80	1740.71	-455.06
28.50	912.55	600.00	82.20	0.58	940.87	959.91	955.78	15.45	6.99	1774.52	-463.00
29.00	911.09	600.00	82.20	0.58	940.49	959.91	955.69	15.45	7.19	1808.33	-471.17
29.50	909.64	600.00	82.20	0.58	940.09	959.91	955.61	15.32	7.38	1840.93	-481.35
30.00	908.18	600.00	82.20	0.58	939.68	959.91	955.52	15.51	7.58	1874.10	-494.54

Figure C-2 Portion of a typical output; the tabulation continues through t = 4000 seconds.

APPENDIX D Program verification

By artificially causing the program PIPETEM to operate with constant values of diffusivity, conductivity, and surface heat transfer coefficient, it may be used to solve problems for which "classical" solutions are known. The program was initially verified by taking O.D. = 10000., W.T. = 1., and comparing results with data points taken from graphs given by McNeill and Brock (12). When a number of such comparisons checked within the accuracy with which the graphs could be read, a more exacting test was made. This is described in what follows.

Özişik (13) gives solutions involving true cylindrical geometry. For a uniform cylinder with inside radius a and outside radius b, insulated on its outer surface, and convectively exposed, with constant surface heat transfer coefficient h, to an internal fluid having "ramp" temperature $\phi = At$ (where A is a constant and t denotes time), and having initial temperature distribution $T(r,0) = 0$ at time $t = 0$, the solution may be written as

$$T(r,t) = \sum_{m=1}^{\infty} \tau_m(t)\psi_m(\rho)/\Delta_m\psi_m(1)$$

where

$$\tau_m(t) = 2AB\alpha[\exp(-\gamma_m t) - 1 + \gamma_m t]/a^2$$

$$\gamma_m = \alpha(\mu_m/a)^2$$

$$\psi_m(\rho) = Y_0(\rho\mu_m)/Y_1(\bar{\rho}\mu_m) - J_0(\rho\mu_m)/J_1(\bar{\rho}\mu_m)$$

$$\Delta_m = \gamma_m{}^2\{[\bar{\rho}\psi_m(\bar{\rho})/\psi_m(1)]^2 - 1 - (B/\mu_m)^2\}$$

$B = ah/k$ (Biot's number), $\rho = r/a$, $\bar{\rho} = b/a$

and the numbers μ_m are the zeros of the function

$$[BJ_0(\mu) + \mu J_1(\mu)]Y_1(\bar{\rho}\mu) - [BY_0(\mu) + \mu Y_1(\mu)]J_1(\bar{\rho}\mu)$$

The symbols J_i and Y_i denote Bessel functions.

It proved to be quite difficult to devise a program to perform these calculations. An apparently different solution, given by Luikov (10), was dealt with first, but debugging was not completed successfully. A later work by Luikov (11), in Russian, gave the form we actually used, citing Özişik (13) as source. Both single and double precision versions of an implementing program were constructed and debugged. The double precision version, much slower than its counterpart, partly because of the use of a "home-made" DP Bessel function subroutine, was used to verify that the single precision version provided adequate accuracy. In all such checks, results from the two programs agreed to five significant digits.

The test problem employed to check the integrity and accuracy of PIPETEM was as follows: a = 4 inches, b = 5 inches, A = 1., B = 1., α = 1., k = 1., k' = 0., h = 0.25. Analytical calculations were made for t = 0, 1., 2., 3., 4., and 5. PIPETEM calculations were made for t = o. to t = 5. at intervals of 0.1. In PIPETEM we took N = 10 and NIT = 5. (We wasted a few seconds by not usinf NIT = 1. This problem is completely linear and no iteration is necessary.) In the analytic solution initially we took account of ten terms in the summation.

The results were so close that differences can not be displayed graphically. However, the differences were sufficiently great as to demand an explanation. Truncation error in the analytic solution was suspected. Accordingly, the analtic solution was repeated three more times using five, twenty, and forty terms. (The forty term solution was very extravagant of computer time.) Plotting typical analytic

results versus reciprocal number of terms clearly indicated that indeed the analytical solution did suffer from truncation error and that extrapolation did indeed lead to the result given by PIPETEM. Since the theoretically "exact" solution has by far the greater error, it is not easy to estimate the accuracy of the PIPETEM results, but in this problem the error appears to be only little more than one part in one thousand. The following tabulation gives calculated results for T(0,5), for example.

N = 5	N = 10	N = 20	N = 40	N = ∞	PIPETEM
2.05694	2.08627	2.09991	2.10647	2.11496	2.11220

Table D-1 Calculated results for T(0,5)

The first four tabulated results were obtained by the single precision analytic solution. (The double precision version also gave 2.08627 for N = 10.) The extrapolated value, for N = ∞, was obtained by assuming that $T = A + Bx + Cx^2 + Dx^3$, where $x = 5/N$, leading to the evaluations: A = 2.114958096, B = -.073526667, C = .049093333, D = -.033584762. The difference between the extrapolated analytic value and the value provided by PIPETEM is 0.00275 which is 0.13% of the average of these values.

As an ultimate check on the validity of the program, an artificial nonlinear problem was made up and solved numerically. The conditions are: inside radius a = 1, outside radius b = 2, conductivity k = T (so that k' = 1), heat transfer coefficient $h = 15e^t$, fluid temperature $\phi = 6e^t$, initial temperature distribution $T(r,0) = r + 4/r$. The diffusivity α is a function of both temperature and time, viz:

$$\alpha = Y/[1+(4-Y)^2/(4+Y)^2],\ Y = (Z-\sqrt{Z^2-16})^2/4,\ Z = Te^{-t}$$

It is easy to verify that $T(r,t) = e^t(r + 4/r)$ is a solution of this problem. Although matters of uniqueness of solution to nonlinear systems are difficult to deal with, it happens that PIPETEM indeed yields the solution cited.

Attention was focussed on the values $T_A = T(1,t^*) = 100.000$ and $T_B = T(2,t^*) = 80.000$, where $t^* = \log_e 20$. Percent errors are shown in Table D-2 corresponding to the 128 evaluations indicated, values of r being 1, 2; values of N being 2, 4, 6, 10; values of NIT being 1, 2, 5, 10; and values of DTEE being t*/5, t*/20, t*/50, t*/200.

DTEE	NIT	N = 2	N = 4	N = 6	N = 10
t*/5	1	-21.51/-14.04	-14.70/-7.173	-9.532/-2.487	-3.372/+3.679
	2	-2.726/+1.479	+.3159/+3.258	+.0102/-.0009	-.7222/-2.646
	5	-2.592/-6.721	-1.784/-6.351	-.9409/-3.697	+1.289/+3.447
	10	-.8019/+4.055	+.5816/+3.232	-.0216/+.3814	-2.023/-6.768
t*/20	1	-2.042/+2.726	+1.085/+4.366	+1.840/+4.425	+2.569/+4.810
	2	-1.962/-.9932	-1.070/-1.907	-1.010/-2.239	-1.104/-2.594
	5	-1.648/+.8822	-.3848/+.7397	-.1063/+.5389	+.0671/+.5541
	10	-1.735/-.1605	-.5519/-.0690	-.2782/-.0115	-.1289/-.1902
t*/50	1	-1.379/+2.520	+.2108/+2.209	+.6620/+2.185	+.9698/+2.209
	2	-1.840/-.1536	-.7005/-.5568	-.4543/-.5960	-.3513/-.6870
	5	-1.714/+.4576	-.5162/+.1779	-.2334/+.1084	-.0781/+.0837
	10	-1.723/+.3710	-.5285/+.0778	-.2466/-.0069	-.0916/-.0043
t*/200	1	-1.631/+1.020	-.3372/+.7247	-.0229/+.6389	+.1658/+.6182
	2	-1.734/+.3131	-.5426/+.0234	-.2618/-.0232	-.1089/-.0567
	5	-1.725/+.3715	-.5287/+.0857	-.2464/+.0365	-.0910/+.0131
	10	-1.725/+.3713	-.5288/+.0489	-.2465/+.0351	-.0911/+.0114

Table D-2 Percent errors in evaluation of T_A and T_B in test problem. Error in T_A (T_B) appears to left (right) of slash (/).

It may be noted that the accuracy is quite sufficient for engineering purposes if the number of layers N = 4, the number of iterations NIT = 5, and the total time interval is divided into 20 parts. It may also be remarked that the total time to evaluate all 128 results was only 2 minutes 20 seconds.

APPENDIX E Heat Transfer at Outer Radius

Following the completion of the preceding portions of this report, an application came to attention which called for the augmentation of PIPETEM capabilities. Ordinarily high temperature piping and fittings are lagged (insulated). However, in some cases it might be of interest to follow the temperature history of unlagged pipes when subjected to internal fluid transient temperatures. Also, PIPETEM is obviously applicable to insulated flanges if one discounts axial conduction and the additional thermal resistance of the flange-bolt interfaces. Down-transients (i.e., decreases in the temperature of fluid contents) lead not only to local thermal stresses but also to discontinuity thermal stresses if there are significant changes in wall thickness, since the average temperature of the thicker part lags (in time) behind that of the thinner part. This is particularly true of flange-pipe assemblies. Leaving the flange unlagged (but with lagging on the pipe itself) causes the average temperature of the flange to be lower than that of the pipe since the former can dissipate heat by convection and radiation. Then, when a down-transient occurs, the resulting thermal dislocation stress will be less than what it would have been if the flange had been lagged. The calculation of the temperatures in an unlagged flange really calls for finite element treatment since axial conduction and heat loss from bolts, nuts, and flange face is obviously of importance. However, a useful estimate of the effect can be gained by considering only radial flow, with, perhaps, an artificially enhanced value of the effective convective

external heat transfer coefficient.

Accordingly, PIPETEM has been modified to account for a convective condition at the outer boundary, viz.

$$k_M \beta = h(\psi - T_M) \qquad \text{E-1}$$

rather than $B = 0$. The Mth finite difference equation becomes

$$\{\beta_M + 2 + \sigma^+[2 + \frac{1}{n_2} - \rho_M^+\sigma^+(T_M^+ - 2\psi^+)]\}T_M^+ - 2T_N^+$$
$$= (\beta_M - 2)T_M + 2T_N - \sigma(T_M - \psi)[2 + \frac{1}{n_2} - \rho\sigma(T_M - \psi)]$$
$$+ \sigma^+\psi^+(2 + \frac{1}{n_2} + \rho_M^+\sigma^+\psi^+) \qquad \text{E-2}$$

This equation, which takes the place of equation 16 and which reduces to equation 16 when h (external) = 0, may be compared term by term with equation 14. In equation E-2, quantities have the meanings ascribed to them on page 5, but the evaluations are at the outer radius. Note that all signs are the same (as in equation 14) except for the $\frac{1}{n}$ terms. The quantity ψ is the external ambient temperature (for convection).

External heat transfer can take place both by convection and by radiation, so that the coeffcient h in equation E-1 is the sum of two coefficients

$$h = h_c + h_r \qquad \text{E-3}$$

The first, h_c, is the actual convective coefficient. We have presumed natural convection from a cylinder into still air, using the relation

$$h = 0.24(\theta/d)^{0.25} \qquad \text{E-4}$$

on page 218 of reference 8. Here

$$\theta = |T_M - \psi| \qquad \text{E-5}$$

and d is the diameter of the cylinder, i.e., the outer diameter of the pipe. The "radiative-convective" coefficient h_r is an equivalent convective coefficient to account for radiative heat transfer and is de-

fined by the equation

$$Q = \sigma\varepsilon A[(T_M+C)^4-(\bar{\psi}+C)^4] = h_r A(T_B-\psi) \qquad E\text{-}6$$

in which σ is the Stefan-Boltzmann constant (0.1714×10^{-8} Btu/hr ft^2 R^4), ε is the emissivity (we take 0.8 for bare steel), A is area (which cancels), $\bar{\psi}$ is the temperature of the surroundings with which radiative heat transfer takes place (this may be and usually is different from the convective ambient temperature ψ), and C = 460 is the constant required to convert Fahrenheit temperatures to Rankine (absolute) temperatures.

The value of h determined from equation E-3 is in units of feet and hours rather than inches and seconds and it must be multiplied by $1/(720)^2$ before being used in PIPETEM. There is provision in PIPETEM for using the relation

$$h = f_c h_c + f_r h_r \qquad E\text{-}7$$

where f_c and f_r are "augmenting factors" which may be used to account for the fact that the geometry is not exactly that of an infinite cylinder so that a "face" or protuberances such as nuts and bolts may convect and/or radiate more heat than would otherwise be accounted for.

The convective ambient temperature ψ and the radiative ambient temperature $\bar{\psi}$ could be made to be functions of time, just as with the internal fluid temperature ϕ, but this has not been done in the present implementation in which ψ and $\bar{\psi}$ are taken to be constant.

The input (and the input instructions) have been modified slightly to accommodate this added facility. For each problem an additional card is inserted after the first card described previously. If this card is blank, perfect insulation is assumed at the outer surface. Otherwise, the card inputs ψ, $\bar{\psi}$, and ε.

Another slight modification has also been made in PIPETEM. In the first card of a problem deck, setting TINIT = -500. causes the program to search for and read initial temperature data as described earlier, while setting TINIT to any value greater than -485. causes the program to assume a constant initial temperature equal to the value of TINIT. The new capability is that setting TINIT = -490. causes the program to start with initial temperatures equal to the final temperatures for the preceding problem. Unless the values of N are equal for the two problems, some nonsense is generated. In use, of course, this capability is intended for a succession of two or more problems involving the same pipe, divided into the same number of layers. A modified set of COMMENT cards at the beginning of the program describes the modified capabilities and the details of the problem deck.

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